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LP2I – Lycée Pilote
Innovant International

A MYSTERIOUS CUP

A cup or a glass is able to produce sounds that are sometimes strange, as the one you can hear when you strike a cup that contains hot milk and in which you added powder chocolate. The purpose of this project is to understand the origin of this sound. In this report, we present the process we followed to achieve it.

I - A curious phenomenon

1) Description of the phenomenon

Let's take a cup, pour some milk in it and put it in the mircowave so that the milk is warm. Add a spoon of chocolate powder and stir the whole mix for a little while. Finally, use the spoon to repeatedly knock on the bottom of the cup. (We filmed the experiment, you can watch it following this link:

http://www.dailymotion.com/video/x5ap0g6_les-mysteres-de-la-tasse-phenomene-etudie_school)

From that knocking, we can hear a series of sounds where frequencies become more and more high-pitched as time goes by (first figure).

We need to explain the reason of this frequency variation.

2) What are the reasons for the increase in sound frequency?

Since the frequency varies throughout the time, it's because another parameter also varies. It's this parameter that we need to find in order to explain the evolution of the frequency. Then, after doing this mix what can happen over time?

a - Is it due to a temperature variation?

Our first idea was to look at in temperature variation. To see if the temperature was the reason of the evolution of the frequency F, we wanted to compare temperature T's evolution and frequency F's evolution to see if they were correlated.

The figure number 2 shows the evolution of the temperature that we measured. The decrease of the temperature is normal. That being said, we turn the curve of the temperature evolution upside down, that way, the comparison between T and F's evolution was more obvious. (we simply made sure that the scale of the abscissa axis (the time) was the same for the two superposed curves)

The third figure shows the result of the curves superposition. We can see that the temperature's evolution has nothing to do with that of the frequency, that way, we can confirm that the temperature's evolution is not the parameter that is going to influence the evolution of the frequency that we are trying to explain.

b - Can the phenomenon be observed with another mix?

Since we were starting to run out of ideas to explain this phenomenon, we looked for inspiration by doing other experiments: we first tried to change the liquid poured in the cup, then we tried to change what we added to this liquid. It turned out that we obtained this same phenomenon mixing sugar, honey, and coffee with milk, only if the milk is hot/warm. We then poured the same ingredients into water, and then again, the phenomenon happens in warm water. The advantage here is that by pouring a little bit of coffee solution into the water, we get a really clear mix.

This led to the formation of microbubbles that came up to the surface and ended up disappearing. It seems like it's during this “going-back-to-surface” process that the frequency of the sound increases. We now had a new lead to explain.

3) Are bubbles the key to the mystery?

a) Is the evolution of sound frequency linked to the presence of bubbles?

To answer this question, we needed to be able to compare the evolution of bubbles and the evolution of the frequency. We had the idea to launch a laser beam through the beaker and to follow the evolution of light intensity emerging from the beaker. Indeed, the higher the number of bubbles is in the beaker, the more diffused the laser light is, as a result the transmission of the intensity is greater.

To recover as much data as possible, we had the beaker diffused by a set of lasers beams capable of following the evolution of the concentration of bubbles according to several directions at the same time.

Experimental device:

We used a horizontal laser beam (figure 4) that we sent on a system to have an output of several laser beams. We placed the network in the focal foyer object of one convergent lens so that the beams penetrate the beaker horizontally. On the other side of the beaker, we positioned photoresist allowing to follow the evolution of the passed on light intensity. We also had the contents of the beaker penetrated by a set of vertical laser beams by using the same type of assembly. On the other hand, during this experiment, we ceaselessly hit the cup to measure the evolution of the frequency. Finally, we had to generate bubbles in the water contained in the beaker. For that purpose, we were lucky to have a faucet which releases heavy gas-filled water, and in which big quantities of microscopic bubbles appear immediately after the water is poured into the beaker.

Results and commentaries: the video of the experiment is available by using the following link

http://www.dailymotion.com/video/x5ap0g4_influence-des-bulles_school

These are the obtained results (figure 5). The maximum intensities of the different curves are identical because we normalised them. The 4 curves evolve then the same way. The moment when they start to grow isn't the same for each curve because the bubbles start to disappear from the bottom of the beaker. It's then totally understandable that the blue curve, which represents the horizontal laser beam, passes through the bottom of the beaker, therefore is the first to grow.

We superimpose the sound frequency to these curves (figure 6), by insuring that the time scale stays the same so the superposition is significant. The results are without doubt: we notice that the evolutions are similar whatever the considered laser beam. The experiment shows that the evolution of sound frequency is undoubtedly linked to the bubbles formed in the water. However, we can still try to learn more: do the bubbles influence the sound frequency throughout their concentration? Can their radius have an importance?

b - Influence of the bubble's size.

We met researchers of Poitiers with who we discussed about our project. They explained us in particular that the phenomenon which causes an attenuation of the light intensity when there are bubbles is the Mie scattering. So, we looked for information about this scattering. We learned in particular that the light diffused during the Mie scattering is not isotropic. The bigger the bubbles, the more forward the scattering. The beam is diffused in a shape characterized by an angle linked to the bubbles' size. This gave us an idea:

If we measure the angle of diffusion over time in the water from the tap, it might be possible to see if the bubbles' size changes. And if we compare the evolution of the angle of diffusion with the frequency evolution, we can conclude that maybe the bubbles' size plays a role in the evolution of the sound frequency. So, we measured the variation of the angle of diffusion over time (figure 8) and realized a video visible following this link:

http://www.dailymotion.com/video/x5ap0g5_influence-taille-des-bulles_school

As shown by the curve (figure 9), the diffusion angle is quite constant, before collapsing in about 5 seconds only. The frequency evolution is slower, we can see on the graph of figure 8 that it spreads in about 30 seconds. We can conclude that the bubbles' size doesn't seem to be the parameter that influences the evolution of the frequency the most. The brutal collapse of the diffusion angle is only linked to the time of the disappearance of the bubbles.

c- Therefore would the key parameter be the concentration of the bubbles?

We know that the evolution of the sound frequency follows the evolution of the intensity of the transmitted light of the LASER perfectly. Now we must prove that the transmitted intensity of the LASER is actually linked to the concentration of the bubbles in the container.

But how can we do this?

We simply decided to count the bubbles over time and to compare the evolution of the number of bubbles to that of the intensity of the transmitted light.

How? With the help of the machine (figure 10) composed of a Smartphone and of a convergent lens.

If we place the lens of the smartphone so that it is superposed on the convergent lens, and using the digital zoom of the camera, we get a machine able to grow a clear observable zone 50 times (of which we evaluated the dimensions with the help of the grid and of the 2 threaded rod beforehand calibrated).

The video of the experience is available following this link:

http://www.dailymotion.com/video/x5ap0g3_influence-concentration-bulles_school

We sent a Laser beam to go through the clear observable zone, the idea was to compare the intensity of this light to that of the evolution of the concentration of the bubbles at the same height of the container.

When the machine was ready, we filled the container with the water containing the microbubbles and we filmed the bubbles of the observable zone with the smartphone. Figure 11 shows the obtained results:

The obtained movie lasts about 1 minute. After applying filters on the video, for the bubbles to be more distinguishable, we could count the bubbles, each second of the film. Figure 12 shows this counting. On the right diagram, we returned the curve showing the evolution of the frequency over time, always taking care to respect the temporal scale of the axis of the abscissas. The superposition of these two curves is very spectacular! The result shows that the sound frequency is connected to the concentration of the present bubbles in the liquid.

But how are the bubbles able to modify the sound frequency produced by the cup?

4) How can the bubbles modify the sound produced by the blows on the cup?

a) Hypothesis

Our first idea was to suppose that the bubbles modify the way the waves propagate. Indeed, we know that the sound wave propagates at a speed of about 1500 m/s (meters per second) in water, whereas it is just 340 m/s in the air, so in the bubbles. Therefore, as the bubbles disappear from the beaker the sound waves propagate with increasing celerity. On the other hand, the wavelengths of the cleans vibrations modes are linked to the geometry of the system. However, in the case of the cup, the presence of bubbles does not change the geometry of the system. Thus, the wavelengths of the cleans vibrations modes of the cup of water, with or without bubbles, must not vary.

Now, it is known that the celerity v, the wavelength and the frequency of a wave are connected by the relation: $v = \lambda \times f$

Therefore, given that λ is constant, if the celerity increases when the concentration of bubbles decreases, then the natural frequency of vibration must also increase while the bubbles disappear. This is what we see experimentally.

But how can we verify that this is what influences the evolution of the frequency?

b - Experimental verification

To verify the hypothesis formulated in the previous paragraph, we had the idea to place an ultrasonic transmitter and receiver in the water with bubbles, facing each other, separated by a few centimetres, in a quite large circular container (figure 13). The transmitter then sends a sinusoidal signal of constant frequency. Without bubbles, we imagine that the signal received by the receiver is in phase with the signal emitted by the transmitter. With bubbles, since the velocity of the waves is altered, and we still have the relation $v = \lambda \times f$, the wavelength of the wave propagating between the emitter and the receiver is also modified (since this time, with this reasoning, it is the frequency of the wave which is constant), and the signals transmitted and received then no longer have any reason to be in phase. This makes it possible to measure the extra time taken by the disturbance to go from the transmitter to the receiver over time in the presence of bubbles. Then, knowing the initial distance between these two sensors, we can then go back to the celerity of the waves when the bubbles disappear. The video of the experiment can be seen by following the link:

http://www.dailymotion.com/video/x5ap0g2_influence-celerite-avec-les-bulles_school

Figure 14 shows the results obtained. The diagram on the right shows the superposition of the evolution of celerity and the typical evolution of the frequency that we measured in a beaker. We have again respected the time scales. It should be noted that the celerity follows very closely the evolution of the frequency.

The superposition of the two curves seem, at first sight, to validate our hypothesis: the bubbles modify the celerity of the waves, which modifies the natural frequencies of vibration of the water's cup.

But in reality, there is a quantitative problem:

From the phase difference and the distance between the transmitter and the receiver, it was possible to measure the celerity of the wave in the presence of bubbles. At the beginning of the experiment it is about 200 m / s, whereas it is 1500 m / s when there are no more bubbles. The celerity is therefore multiplied by 7.5 during the experiment. Now the frequency only goes from 1000 to 1600Hz, which is not compatible with the relationship $v = \lambda \times f$.

To conclude:

Bubbles are responsible of the evolution of the frequency, but the variation of speed of waves in the water doesn't explain the variation of the frequency.

That means that the waves in the water don't have any role into the evolution of the frequency. The waves we're interested in must be contained in the material of the cup.

But how the presence of bubbles can modify the vibration frequency of the beaker if the waves are confined in the material that compose the beaker?

We need to know more about the way the beaker vibrates. And we also need to provide an update: what is true in what we wrote, and what can be contested?

II - Back on the way a cup vibrates

1) How a cup vibrates ?

By tapping on the cup that contains water, and by examining (Fourier analyze) emitted sound, we can see that the sound contains multiple frequency spikes. The cup of water vibrates following multiple modes of vibration at the same time. However, those frequencies don't match up because they don't follow the relation $f_n = n \times f_1$, where f_1 is the frequency of the basic mode, and fn is the frequencies of the other modes. But we didn't stop at this experiment. Now knowing the frequencies of the modes, we looked forward to make the cup of water vibrates following those modes, thanks to a vibrating membrane that we stuck to the cup. The final result is spectacular! We took a photo of the surface of the water in the beaker (16th figure). On the other hand, the video of the experiment can be seen following this link:http://www.dailymotion.com/video/x5ap5be_vibration-tasse_school.

That way, we observed the standing waves of the 3 first modes of the vibration of the cup. The first mode contains 4 vibration antinodes and 4 nodes, so 2 wavelengths, the second mode contains 6 nodes and 6 vibration antinodes, so 3 wavelengths, the third one contains 8 nodes and 8 vibration antinodes, so 4 wavelengths.

We divided the perimeter by 2 to have the value of λ_1 for the first mode, by 3 to have the value of λ_2 for the second mode, and by 4 to have the value of λ_3 for the third mode. Knowing the frequencies of the three modes and the wavelengths, we calculated the speed of the waves for each mode (table 17). We then got surprised by two things: we found that the values of the speed were weak, and we were particularly surprised to see so much different speeds depending on the mode, when we thought finding a constant speed, as it's the case for the different modes of variation of a guitar cord. L

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But finally, the weak values of the speeds translate the fact that those waves are not waves, of compression, but waves of flexion. On the other hand, the fact that speed of those waves depend of the frequency shows the property of this glass for this type of wave, property that we cannot see in the guitar chord part. It's the reason why the modes of variation of the beaker don't follow the relation $f_n = n \times f_1$!

After reflection, we are convinced that the modes 1, 2 and 3 that we observed are in fact modes 2, 3 and 4

The mode 1 corresponded to a perturbation such as we have just one λ for all the perimeter of the beaker. So why do not we see mode experimentally? We tried to answer this question by making diagrams: Mode 2 has four nodes and four antinodes. Mode 1 is expected to present two nodes and two antinodes. The diagrams here show the beaker seen from above undisturbed (in green) and disturbed (in red) for mode 2 (left diagram), and for mode 1 (right diagram) (picture 18).

As the perimeter of the beaker must keep the same value, the forms green and red must measure the same length.

So, on mode 1, the vibration of the beaker must be such that the left side of the glass should be closer to the center of the circle so in the same way that the right moves away, which simply amounts to moving the top of the beaker to the right, than the base of the beaker does not move. Then this form, very asymmetrical of the beaker, would not have a great stability. We think this is the reason why this first mode is not visible.

2) So, what are we bringing all these new observations?

First of all, it is clear that the waves responsible for the sound do not propagate in the liquid.

It circulates along the perimeter of the beaker. So, it’s the perimeter of the beaker that fixes the wavelengths of the eigenmodes. And secondly these waves are bending waves that push the liquid in the cup. So, it is very possible that the presence of bubble changes the way the cup pushes the liquid.

Now we believe that bubbles modify the mechanical properties of the liquid, and that is what is causing the change in frequency.

Since the beaker vibrates laterally, the wall of the beaker, vibrating, Pushes a certain mass of water. We could then model the cup of water with a "solid spring" device.

In this modeling, the beaker vibrates with a certain elasticity, which would be represented by the spring constant k of the spring and the mass of the water corresponds to the mass attached to the spring end. In accordance with the equation giving the oscillation frequency of such a device: $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, more the mass is large, and more the beaker glass oscillates with a low frequency.

We wanted to see if this modeling was possible quantitatively. In order to do this, we measured the vibration frequency of the cup's fundamental mode in accordance with the mass of the liquid contained by the cup, while keeping steady the height of the liquid. Therefore, we needed to work with liquids of different densities. Tracing then the frequency in accordance with $1/\sqrt{m}$, we actually get a straight line which confirms the modeling (picture 19). This said, used liquids didn't have varied densities on a sufficiently wide area to really confirm the modeling in a quantitative point of view.

But this modeling stays valid in a qualitative point of view. And this modeling on which we are insisting on is then essential to understand the action of the bubbles on the frequency's evolve.

3) Vibration of the beaker in the presence of gas bubbles: denouement

As we said, we are convinced that mechanical attributes of the liquid are modified by the presence of bubbles. Which mechanical characteristic of the liquid is then modified in the presence of bubbles?

In the expression $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, the coefficient k represents the cup's elasticity, and m the mass of water. More the environment is easily deformable, and more k is low. But in reality, since the cup is vibrating even without water, it's because the mass of the cup is included in the term “m” of the formula. Likewise, we have been able to show that 2 liquids with identical masses but with different densities produced sounds with different frequencies. Therefore, the liquid's elasticity acts in the coefficient k as well.

Let's return on the formula $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. More k is high, more the vibration frequency is high. As in our results, we can see that the presence of bubbles makes the vibration frequency reduce, we can think that the vibrating system cup + water possesses therefor a lower coefficient k in presence of bubbles. And this can be understood easily because, justly, the presence of bubbles makes the environment more compressible, so more easily deformable. Therefore, k is lower in presence of bubbles! In the end, the system's elasticity, which sets the vibration frequency of the system, would results so from a coupling between the cup's elasticity and the liquid's elasticity contained by the cup.

Can we prove it?

Further to a meeting with some researchers of the CINAM in Marseille, we tried to verify through experimentation this hypothesis, producing by electrolysis hazes of bubbles in directions connecting loops, then nodes. Hazes of bubbles in directions connecting nodes mustn't modify the frequency, contrary to hazes of bubbles connecting loops. In a first time, results didn't show any difference. So, we produced more bubbles using springs instead of simple wires. We began then to have a difference of performance measurable, in the order of the ten of hertz, proving the significance of the liquid's compressibility containing the bubbles.

The project we have carried out has helped us to understand the origin of the change of the frequency we can hear when a tee spoon struck a cup that contain hot water and in which we added chocolate powder.

This evolution of the frequency is due to the occurrence of gas microbubble in the liquid, that change the elasticity of the system “cup + liquid”, and then the natural frequencies of the vibration of the cup.

We were passionate about this project because each step was a riddle to solve that required experiment to find the solution.

Finally, at the beginning, in this project, we firstly thought this project was only a riddle, and we thought we just were looking for a solution about our riddle. But our work can go farther than that.

Indeed, we have been contacted by engineer of the laboratory SAINBIOSE. This laboratory works about health, biology and engineering. It’s possible that our results can be used in their research work.

We feel honored and very enjoy to participate in this exciting partnership. Many way can be followed. For example, it’s possible to inject microbubble in the flow of the liquid to change the elasticity of the membrane in which the liquid flows.

Finally, we can say today that the end of this story is not written. And it’s with real pleasure we will continue to do this physic, and not only at the breakfast!

Thanks to our partners, for their support, their collaboration, and for the interest in our work.